# POPULATION INVERSION AND RADIATION DENSITY IN A Q-SWITCHED $CO_2$ LASER

P. A. Skovorodko and Yu. A. Yakobi

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The mechanism of lasing in a steady-state $CO_2$ laser has been investigated in [1-3]. In this paper we present a numerical analysis of the processes which occur in a $CO_2$ laser when the resonator is rapidly Q-switched. It is shown that the transition of the laser into the state with a new Q has an oscillatory form.

1. Formulation of the Problem and Method of Calculation. The processes which occur in a laser can be described by a system of kinetic equations for the populations of the laser levels and the radiant energy density in the resonator [4, 5].

A specific feature of gas lasers is the change in the populations of the energy levels due to inelastic scattering with other atoms (molecules). Taking this into account, and at the same time neglecting the nonuniformity of the medium and field, we can write the system of kinetic equations for a Q-switched gas laser in the form

$$\begin{aligned} \frac{dN_2}{dt} &= V - W_2N_2 - \frac{Bs}{\Delta\nu}(N_2 - N_1)\rho \\ \frac{dN_1}{dt} &= W_2N_2 - W_1N_1 + \frac{Bs}{\Delta\nu}(N_2 - N_1)\rho \\ \frac{d\rho}{dt} &= \left[h\nu\frac{Bs}{\Delta\nu}(N_2 - N_1)\frac{l}{d} - L\frac{c}{d}\right]\rho \end{aligned} \tag{1.1}$$

where $N_2$ and $N_1$ are the populations of the upper and lower laser levels, corresponding to the vibrational states $00^01$ and $10^00$, $\rho$ is the energy density in the resonator, V is the rate at which the upper level is filled, $W_2$ and $W_1$ are the probabilities of deactivating molecular collisions for the upper and lower levels, respectively, B is the Einstein coefficient for stimulated emission from the upper to the lower level, $\Delta\nu$ is the linewidth of the spontaneous transition $2 \rightarrow 1$, L are the relative losses for a single passage across the resonator, c is the velocity of the light, d is the resonator length, and $l$ is the length of the discharge tube. The quantity s represents the relation between the overall population of the vibrational levels and the population of the individual vibrational–rotational levels. Since rotational relaxation occurs fairly rapidly (in a time of the order of $10^{-7}$ sec [1]), the energy distribution between the rotational levels can be assumed to be a Boltzmann distribution with the gas temperature. This distribution is given by the expression

$$n^J = \frac{2NhcA}{KT_0}(2J+1)\exp\left[-\frac{Ahc}{KT_0}J(J+1)\right]$$

where J is the rotational quantum number, h is Planck's constant, $T_0$ is the gas temperature, and A = 0.394 $cm^{-1}$ is the rotational constant for the $CO_2$ molecule. We will henceforth consider only the most intense laser transition between the states $00^01$ (J = 21) and $10^00$ (J = 22). The change in the population for a single rotational quantum is approximately 2%, so that we can write [6]

$$n_2^J - n_1^{J+1} = (N_2 - N_1)s$$

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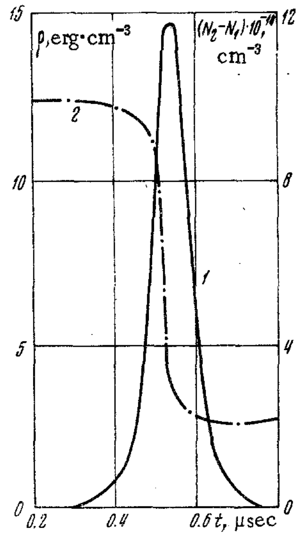


Fig. 1

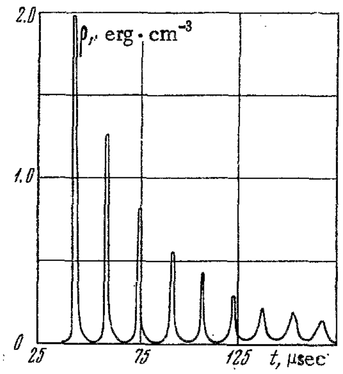


Fig. 2

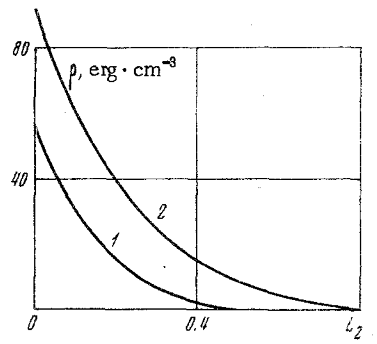


Fig. 3

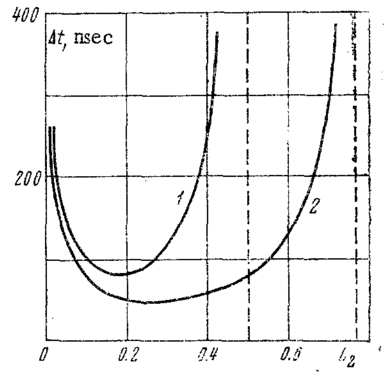


Fig. 4

where

$$s = \frac{2hcA}{KT_0}(2J+1)\exp\left[-\frac{Ahc}{KT_0}J(J+1)\right]$$

If $T_0 = 600°C$, we have $s = 5.3 \cdot 10^{-2}$.

In the set of equations (1.1) we have taken into account only the main processes which fill and empty the levels. This limitation is based on concrete estimates of the comparative contribution of the large number of processes. Filling of the upper laser level occurs as a result of a two-stage process, namely, electron excitation of vibrations of the nitrogen molecules and subsequent energy transfer by collisions to the $00^01$ vibrational level of the $CO_2$ molecule. The excitation probability of the vibrational levels of the nitrogen molecule $W_3 = k_3 n_e$, where $n_e$ is the electron density and $k_3$ is the rate constant of the process

$$N_2(v=0) + e + KT \rightarrow N_2(v=n) + e$$

Using the numerical data in [1-3], we will put $k_3 = 4 \cdot 10^{-9}$ $cm^3 \cdot sec^{-1}$ and $n_e = 5 \cdot 10^9$ $cm^{-3}$. Hence we obtain $W_3 = 20$ $sec^{-1}$. The rate constant of the second stage $k_4 = 7.5 \cdot 10^{-15}$ $cm^3 \cdot sec^{-1}$ [3]. In the $CO_2$ laser one usually uses the following partial pressures: $CO_2$, 1 torr; $N_2$, 2 torr; and He, 5 torr. The probability of the excitation of antisymmetric vibrations of the $CO_2$ molecule is $W_4 = k_4 N_a$, when $N_a$ is the density of the nitrogen molecules. If $N_a = 6.2 \cdot 10^{16}$, we have $W_4 = 470$ $sec^{-1}$. Hence, the rate of excitation of antisymmetric vibrations of the $CO_2$ molecule is limited by the rate of electron excitation of the vibrations of the nitrogen molecules. The rate of this process is $V = \alpha n_e N_a k_3$, where $\alpha$ is the average number of vibrational quanta excited by one electron, $\alpha = 3.5$ [3], and $V = 2.2 \cdot 10^{18}$ $cm^{-3} \cdot sec^{-1}$.

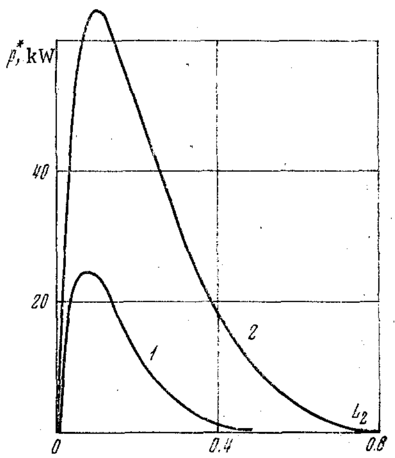


Fig. 5

The upper laser becomes emptied due to induced radiation and collisional relaxation with $CO_2$ molecules. The probability of this relaxation is $2 \cdot 10^3$ $sec^{-1}$ [3]. Relaxation of the lower laser level occurs in two stages – first to the $01^00$ level, then to the ground state. Using the data in [3], we have for the probability of the first stage a value of $5.5 \cdot 10^5$ $sec^{-1}$, and for the probability of the second stage we have $3.8 \cdot 10^{-4}$ $sec^{-1}$. Hence, the rate at which the lower laser level is emptied is limited by the transition $01^00 \rightarrow 00^00$. The width of the spectral line can be assumed to be Doppler. The resonator length (270 cm) and the dimensions of the discharge tube (length 200 cm and internal diameter 24 mm) were chosen to be the same as in [7]. The probabilities of spontaneous radiational transitions from these levels is very small, so they can be neglected.

The nonlinearity of the set of equations (1.1) eliminates the possibility of obtaining an accurate solution even for the simplest variation of L(t). Only when L = const can an analytical solution be obtained. The solution of the set of equation (1) for variable losses was obtained by the Runge-Kutta method on a computer.

2. Fast Q-Switching. We will consider the case when the losses are changed abruptly at the instant of time $t = 0$. For $t < 0$ the losses are higher than the critical values, lasing does not occur, and the population inversion is a maximum. At the instant $t = 0$, the losses fall to a certain value $L_2 < L_*$. The calculated radiation density (curve 1) and the population inversion (curve 2) calculated for the case of the useful loss $L_2 = 0.4$ directly after Q-switching are shown in Fig. 1 as a function of time.

Figure 2 shows the relaxation of the energy density in the steady state, which is a continuation of the graph in Fig. 1 in another time scale (the first pulse is not shown, since its amplitude is seven times higher than the amplitude of the second pulse). The output power P varies in the same way as the energy density, being connected with it by the relation $P = \rho V_0/\tau$, where $V_0$ is the volume of the active medium, $\tau$ is the photon decay time, $1/\tau = (L - L_0)\, c/d$, where $L_0$ are the parasitic losses. At time 0.4 $\mu$sec after Q-switching a stimulated radiation pulse of high amplitude appears, of duration $\sim 50$ nsec. The peak output power is 55 kW.

These characteristics of a giant laser pulse are in good agreement with the experimental data in [7], in which, for a continuous power of 25 W, a giant pulse of duration less than 100 nsec and peak power greater than 10 kW was observed, and the lower time limit was set by the inertia of the detector.

Let us consider in more detail the characteristics of the giant pulse as a function of the useful loss $L_2$. A mathematical experiment was made for two values of the initial losses: $L_1 > L_*$ and $L_1 = 0.5$. The time at which the pulse first appeared increases slightly with $L_2$, and then increases sharply as $L_2$ approaches $L_1$. This is due to the fact that for large losses, the excess of gain above the attenuation becomes less. Note that for $L_1 = 0.5$ a giant pulse occurs more rapidly. The reason for this is that when $L_1 = 0.5$ the radiation density increases from a high value (weak lasing occurs), while when $L_1 > L_*$ the radiation density increases from the spontaneous radiation density. This case is assigned as the initial condition $\rho|_{t=0} = \rho_0$.

The characteristics of the first giant pulse as a function of the useful loss $L_2$ for the cases when the initial losses $L_1 = 0.5$ (curve 1) and for initial losses which exceed the critical losses (curve 2) are shown in Fig. 3 (the radiation density), Fig. 4 (the pulse width $\Delta t$), and Fig. 5 (the peak power $P^*$).

It turns out that the pulse of stimulated emission is greater the greater the difference $L_1 - L_2$ (Fig. 3). The mathematical experiment also shows the presence of a minimum in the pulse width as function of $L_2$ (Fig. 4). Initially, as $L_2$ increases a reduction in the width is observed from 400 nsec for $L_2 = 0.05$, to 50 nsec for $L_2 = 0.25$. This is due to a reduction in the decay time of the photons. However, when $L_2$ is increased further, due to the small excess of the gain over losses, the pulse "flares up" very slowly. It has been noted in [7] that the width of the pulses decreases as the losses increase. The basis of this was an experiment with two different values of the transmittance of the resonator mirror. However, as Fig. 4 shows, a way of reducing the pulse width is by finding the optimum value of $L_2$. The peak output power $P^*$ (Fig. 5) has a maximum as a function of $L_2$, due to the presence of a minimum in the pulse width.

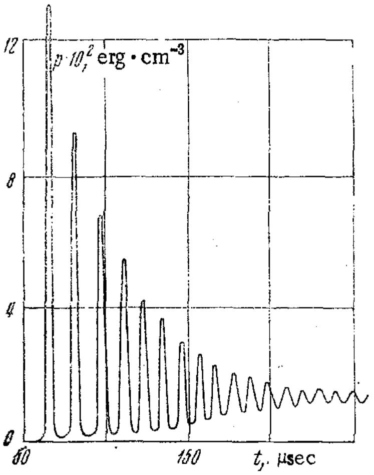


Fig. 6

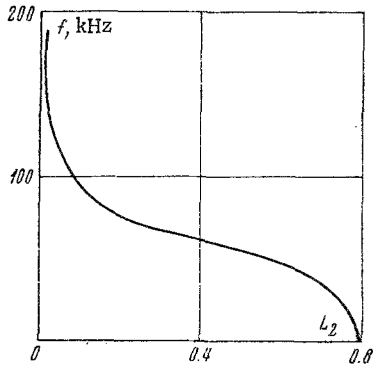


Fig. 7

The power for $L_1 = 0.5$ is considerably less than for $L_1 < L_*$, since in the first case the stored energy is less and the pulse width is greater.

The mathematical experiment shows that following the first pulse there is a whole series of pulses of decreasing amplitude and increasing width, with a repetition frequency of approximately 50 kHz (Fig. 2). The appearance of the second and subsequent pulses can be explained as follows. During the buildup of the first pulse the population inversion falls rapidly, and it continues to fall due to the presence of radiation at the instant when the gain is equal to the attenuation. Hence, at the instant when the first pulse ceases the population inversion falls below the steady-state value, corresponding to the losses $L_2$. After this, the population inversion again begins to increase, and due to the fact that the radiation density lags behind the inversion, it reaches values which exceed the steady-state value, the second pulse occurs, etc.

Obviously, when the rotating mirror method [7] is used to produce Q-switching, the last pulses cannot be observed due to the small Q-switching time. We Q-switched a $CO_2$ laser using a mechanical chopper in the resonator cavity. In this case, after the first giant pulse, we observed a whole series of subsequent pulses at intervals of 10-15 $\mu$sec, of rapidly decreasing amplitude.

3. Fast Q-switch-off. Finally, we will consider the case when the losses are abruptly increased from $L_1$ to $L_2$, but nevertheless remain below the critical values. Figure 6 shows the radiation density $\rho$ as a function of time for the case when the losses are suddenly increased from $L_1 = 0.1$ to $L_2 = 0.2$. In this case the gain become less than the attenuation, as a result of which the radiation density falls so long as the gain and the attenuation are not equal. The relaxation of the radiation density in the steady-state then takes on a pulsed form, and the amplitude of the pulses is then, fairly accurately, an exponentially decaying function of time.

An interesting feature of this relaxation process is the fact that the characteristic repetition frequency of the pulses $f$ is practically independent of the prehistory, i.e., $L_1$, and is a function of the final state $L_2$ only (Fig. 7). This behavior also occurs for the relaxation processes in fast Q-switching (Sec. 2). For a wide range of $L_2$ the interval between the pulses corresponds to the relaxation time of the lower laser level. The overall relaxation time of the radiation density in the steady state depends on the difference $L_2 - L_1$. According to calculations for the $CO_2$ laser, this time varies between 0.1 and 1 msec and corresponds in order of magnitude to the relaxation time of the upper laser level.

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